THE FLOW OF LIQUID IN A STREAM FROM THE STANDARD TURBINE IMPELLER*

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Profiles of the mean velocity have been analyzed in the stream streaking from the region of rotating standard six-blade disc turbine impeller. The profiles were obtained experimentally using a hot film thermoanemometer probe. The result of the analysis is the determination of the effect of relative size of the impeller and vessel and the kinematic viscosity of the charge on three parameters of the axial profile of the mean velocity in the examined stream. No significant change of the parameter of width of the examined stream and the momentum flux in the stream has been found in the range of parameters $d/D \in \langle 0{\cdot}25;\ 0{\cdot}50 \rangle$ and the Reynolds number for mixing $\mathrm{Re}_M \in \langle 2{\cdot}90 \,.\, 10^1;\ 1 \,.\, 10^5 \rangle$. However, a significant influence has been found of Re_M (at negligible effect of d/D) on the size of the hypothetical source of motion — the radius of the tangential cylindrical jet — a. The proposed phenomenological model of the turbulent stream in region of turbine impeller has been found adequate for values of Re_M exceeding $1{\cdot}0 \,.\, 10^3$.

Turbulent flow of a homogeneous or heterogeneous batch mixed by a standard turbine impeller is a result of complex interactions between the charge, the impeller and the baffles. Description of the flow within the whole system depends ultimately on the knowledge of the flow in region immediately adjacent to the region of the rotating impeller which is the source of energy and inertia for the motion in the rest of the charge. In view of the fact that even in the standard arrangements of mixed systems certain quantities vary within certain intervals (geometrical and physical quantities), it is necessary to understand, apart from the mechanism of the flow itself, also the effect of these quantities on the parameters characterizing the velocity field in the mentioned stream.

The problem of the velocity field in a turbulent stream ejected from the region of rotating standard turbine impeller in a cylindrical vessel with radial baffles has been studied experimentally by numerous authors[1–11]. Theoretical solution has been sought for description of this stream as a submerged jet discharging from the region of rotating impeller modelled by a jet[1–7]. Or,

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alternatively, as a pulsating stream with harmonic pulsations induced by the rotating blades of the impeller[11]. None of the cited authors, however, extended his experiments far enough to include the effect of the principal geometrical parameters of the system, and, eventually, the effect of the kinematic viscosity of the charge. Neither did they take into considerations the effect of these quantities on the velocity field in the examined stream. This contribution may thus be regarded as a supplement of the experimental and theoretical studies in the sense of determination of the effect of geometrical and physical characteristics on the parameters of a quantitative description of the mean velocity field in the stream streaking from the blades of the rotating standard turbine impeller.

THEORETICAL

Consider a mixed system with a standard six-blade turbine impeller and a separating disc located in the axis of a cylindrical vessel equipped with radial baffles (Fig. 1). There is a cylindrical frame of reference attached to the system r, z, α_0 whose origin is located at the intersect of the axis of cylindrical symmetry and a horizontal plane passing through the disc. Mean value of local velocity is determined by its magnitude and direction given by the angles α and β. The angle alpha is the angle made by the local mean velocity vector and its projection onto a horizontal plane. The angle β is the angle between the above projection and a straight line through the given point in the radial direction. The quantities $\overline{w}_r$, $\overline{w}_{tg}$, $\overline{w}_{ax}$ are components of the mean velocity vector.

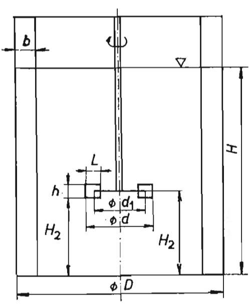

FIG. 1

Mixed System with Standard Turbine Impeller

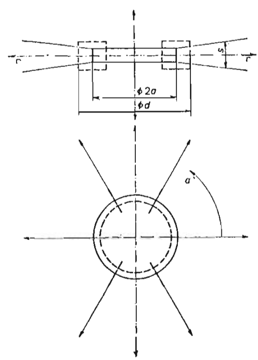

FIG. 2

Cylindrical Tangential Jet

The description of the velocity field in the stream discharging from the space of the rotating blades starts from the equations of continuity and the Reynolds equation[5,6] for the mean motion under the turbulent regime. It is assumed that the impeller as a source of motion may be replaced by an axially symmetric cylindrical slot – the tangential cylindrical jet (Fig. 2). The liquid streaking from the jet has as the only non-zero component the tangential velocity component and enters liquid of infinite extent at rest.

Analytical solution of the above mentioned two equations after introducing eight simplifying assumptions and two suitable boundary conditions has lead to a three parameter equation[5,12,13] expressing the dependence of the radial components of mean velocity $\overline{w}_r$ on the axial coordinate z and, further, to a relation between the radial and the tangential component of mean velocity (Table I). For the purposes of modelling both the dependent and independent quantities are expressed in dimensionless form as

$$W_j = \overline{w}_j/(\pi\, dn), [j = \text{ax, r, tg}]\,; \quad Z = 2z/h\,. \qquad (1a, b)$$

Thus the velocity components are scaled by the peripheral velocity of the outer edges of impeller blades, $\pi\, dn$, and the axial coordinate is measured by the half-width of the blades, $h/2$. The parameter a characterizes the so-called radius of the cylindrical tangential jet (Fig. 2) and may be determined from the knowledge of the angle of the direction of mean velocity, alpha, in the examined stream[12]. The parameters σ (width of the stream) and A (momentum flux in the stream), may be determined from the measured velocity profiles $W_r = W_r(Z)$. The proposed phenomenological model enables also values of the eddy viscosity, ε, to be determined in the stream discharging from the space of the rotating impeller blades (Table I). All mentioned dimensional and dimensionless characteristics may be examined in dependence on the size of the

TABLE I

Characteristics of the Velocity Profile in the Stream Streaking from the Region of Rotating Turbine Impeller

$$W_r = (A/2\pi dn)\,(\sigma/r^3)^{1/2}\,(r^2 - a^2)^{1/4}\,[1 - \text{tgh}^2(h\sigma Z/4r)]$$
$$W_{tg} = W_r \,\text{tg}\,\alpha\,, \quad W_{ax} = 0\,, \quad W = (W_r^2 + W_{tg}^2)^{1/2}$$
$$\alpha = \arcsin\,(a/r)\,, \quad \beta = 0$$
$$\varepsilon = (A/2)\, 1/(\sigma^3 r)^{1/2}\,.\,(2r^2 - a^2)/(r^2 - a^2)^{3/4}$$
$$K_p = \dot{V}/nd^3 = 2\pi^2(A/2\pi nd^2)\,(1/\sigma^{1/2})\,[1 - (2a/d)^2]$$

system, viscosity of the mixed charge and relative size of the impeller and vessel, d/D. It may be assumed that the effect of the geometrical simplex d/D on the values of the above parameters shall be negligible provided the emerging stream is not immediately affected by the walls of the vessel or the baffles, *i.e.* provided the impeller rotates in an "infinite" space. In case of geometrically similar systems it may be concluded that the dimensionless characteristics of the examined stream will not depend on the characteristic dimension either. The effect of the kinematic viscosity, however, shall become manifest unless the value of the eddy viscosity ε is larger in the order of magnitude. The requirement of validity of the proposed model is not then met, namely that the viscous stress may be neglected in comparison with the turbulent stress. Regions of validity of the above statements, however, must be subject to experimental studies in the examined stream.

EXPERIMENTAL

The distribution of mean velocity in the stream streaking from the region of rotating turbine impeller[14] was detected by a hot film wedge-shaped thermoanemometer probe of the DISA type[15]. In preliminary experiments[6] we have assumed negligible axial velocity component $\bar{w}_{ax}$ in the examined stream and found local value of the angle alpha and the absolute value of the mean velocity vector using the hot film probe of the DISA 55 A 83 type. There were always 8 to 12 such preliminary runs on each $\bar{w} = \bar{w}(z)$ profile, while for the given conditions (impeller diameter, frequency of revolution and the kinematic viscosity of the charge) two profiles were usually obtained: One in the proximity of the rotating impeller and the other in a more distant radial position. The total number of detected velocity profiles in the examined stream, N, for given geometrical configuration is shown in Table II; this Table contains a review of the conditions of the all carried out experiments.

TABLE II

Experimental Conditions for Runs with $H = D = 0{\cdot}4$ m; $H_2'/H = 1/2$, $b/D = 0{\cdot}1$; $H/D = 1$;

d/D	Re_M	R	N
0·25	$2{\cdot}56 \,.\, 10^5$	0·02—0·2	2
0·30	$5{\cdot}6 \,.\, 10^1 - 2{\cdot}72 \,.\, 10^4$	0·0214—0·607	8
0·40	$5{\cdot}54 \,.\, 10^1 - 1{\cdot}0 \,.\, 10^5$	0·025 —0·583	12
0·50	$2{\cdot}94 \,.\, 10^1 - 3{\cdot}0 \,.\, 10^4$	0·03 —0·5	8

$d : n : L : d_1 = 20 : 4 : 5 : 15$. Charge: water ($\nu = 1{\cdot}0 \,.\, 10^{-6}$ m^2 s^{-1}), water solution glycerol ($\nu = 2{\cdot}38 \,.\, 10^{-5}$ m^2 s^{-1} and $4{\cdot}62 \,.\, 10^{-4}$ m^2 s^{-1}).

Finally, profiles have been obtained of the quantity alpha and $\bar{w}$ (or W_r) in various radial positions given by the coordinate r and for various geometrical and physical conditions in the system. The $\alpha = \alpha(z)$ profiles (for $r =$ const.) served to calculate, a, from

$$a = r \sin(\alpha_{av}), \tag{2}$$

where α_{av} is the arithmetic mean of measured values of alpha on the given profile; the experimental values of alpha for a given radial coordinate practically did not differ. The quantity $\bar{w}_r$ was then computed for given values $\bar{w}$ and alpha from

$$\bar{w}_r = \bar{w} \cos \alpha \tag{3}$$

and normalized by the peripheral velocity of the rotating blades of the impeller. The axial profiles of the dimensionless radial velocity component were taken in the form

$$W_r(Z) = A_1\{1 - \mathrm{tgh}^2 [A_2(Z - A_3)]\}, \quad R = \text{const.} \tag{4}$$

The dimensionless axial and radial coordinates in Eq. (4) had been defined by Eq. (1b) on the one hand and by

$$R = (r - a)/r, \tag{5}$$

where we had substituted the earlier found value of the parameter a for the given arrangement and viscosity of the charge. The parameters A_1, A_2, A_3 in Eq. (4) were found by nonlinear regression[6] while the goodness of fit of the experimental and computed profiles $W = W(Z)$ was tested statistically (Fig. 3).

RESULTS AND DISCUSSION

Examples of the course of experimental and computed axial profiles of the radial velocity component in the stream ejected from the region of rotating impeller are shown in Figs 3–4; the points represent experimental data, the curves results of nonlinear regression.

The parameter A_1 in Eq. (4) represents maximum radial velocity component of liquid on a given profile normalized by the peripheral velocity of outer edges of impeller blades and may be also expressed (see Table I) by

$$A_1 = [A/(2\pi \, \mathrm{d}n)] \, (\sigma/r^3)^{1/2} \, (r^2 - a^2)^{1/4}. \tag{6}$$

The parameter A_2 represents reciprocal width of the stream on a given position,

$\sigma/(2r)$ normalized by the half-width of the blade, $h/2$, for we have

$$A_2 = \sigma h/4r\,. \tag{7}$$

The parameter A_3 in Eq. (*4*) expresses the dimensionless axial coordinate of maximum on the velocity profile, *i.e.* its shift off the horizontal plane of coordinate $z = 0$. The value of this parameter has been found statistically insignificant (the above described processing of experimental profiles $W = W(Z)$ showed that the parameter, A_3, not to exceed hundredths in the order of magnitude). This corresponds to a position

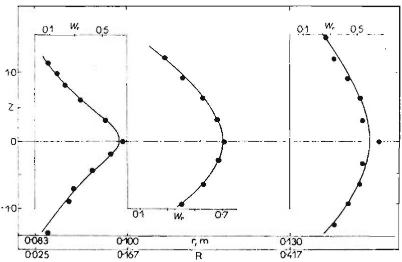

FIG. 3

Typical Experimental Profiles of Dimensionless Radial Mean Velocity Component W_r in the Stream Streaking from the Region of Rotating Turbine Impeller
$d/D = 0{\cdot}40$; $\mathrm{Re_M} = 2{\cdot}94\,.\,10^4$.

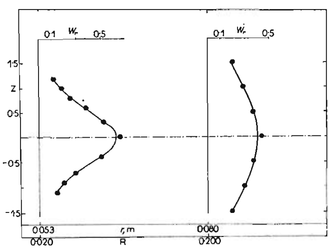

FIG. 4

Typical Experimental Profile of Dimensionless Radial Mean Velocity Component W_r in the Stream Streaking from the Region of Rotating Turbine Impeller
$d/D = 0{\cdot}25$; $\mathrm{Re_M} = 2{\cdot}56\,.\,10^4$.

of the impeller in the geometrical center of the vessel ($H'_2 = D/2$) where the axial profiles of all mean and fluctuation characteristics of the stream streaking from the region of rotating impeller are symmetric with respect to the horizontal plane passing through the separating disc of the turbine impeller.

The values of A_1 and A_2 determined by nonlinear regression were used to calculate, for the known values of the parameter a and the radial coordinate r (or R), the quantity σ (Eq. (7)) and A (or in dimensionless form $A/(2\pi d^2 n)$ (Eq. (6)). Thus we have obtained characteristic parameters of the examined stream to be subjected to further analysis: The quantities σ and $A/(2\pi d^2 n)$ were used to calculate mean values and relative standard deviations for the given geometrical conditions $s_{\sigma \mathrm{rel}}$, or $s_{A/(2\pi d^2 n)\mathrm{rel}}$. Values of these quantities are summarized in Table III. Since the parameter, a, was normalized by the radius of the impeller $d/2$ which eliminates the effect of the d/D ratio on this quantity, only parameters of the following power-law dependence were sought

$$2a/d = C\,\mathrm{Re}_M^c\,, \tag{8}$$

where the Reynolds number for mixing was defined as

$$\mathrm{Re}_M = nd^2/\nu\,. \tag{9}$$

TABLE III

Parameters of the Velocity Field in the Stream Streaking from the Region of Rotating Turbine Impeller

d/D	σ	$S_{\sigma \mathrm{rel}}$ %	$A/2\pi nd^2$	$S_{A/(2\pi nd^2)\mathrm{rel}}$ %	Kp^a	Re_M
0·25	11·86	22·8	0·113	18·2	—	—
0·30	14·81	21·1	0·104	21·2	—	—
0·30	—	—	—	—	0·484	$1{\cdot}0 \,.\, 10^4$
0·30	—	—	—	—	0·552	$1{\cdot}0 \,.\, 10^5$
0·40	13·05	20·7	0·117	19·3	—	—
0·40	—	—	—	—	0·502	$1{\cdot}0 \,.\, 10^4$
0·40	—	—	—	—	0·569	$1{\cdot}0 \,.\, 10^5$
0·50	13·13	16·9	0·111	13·0	—	—
0·50	—	—	—	—	0·472	$1{\cdot}0 \,.\, 10^4$
0·50	—	—	—	—	0·545	$1{\cdot}0 \,.\, 10^5$

[a] $2a/d = \mathrm{Re}_M^{-0.106}$; $\mathrm{Re}_M = 2{\cdot}94 \,.\, 10^1 - 1 \,.\, 10^5$.

The results of the processing of the dependence (8) by nonlinear regression, together with the confidence limits of the quantity Re_M, to which the proposed regression is valid, are also shown in Table III. The results shown in this table may be summarized as follows: All parameters of the velocity profiles in the examines submerged stream are practically independent on the parameter d/D. This means that the stream in the investigated range of d/D values may be regarded as a submerged jet in an infinite liquid. The relatively large scatter of the quantities σ and $A/(2\pi d^2 n)$ about the mean may be attributed to the fact that the above quantities have been found, although not systematically, dependent on the magnitude of the radial coordinate R, in spite that theory does not predict such changes. The significant effect of the Reynolds number on the magnitude of the radius of the cylindrical tangential jet points out the limitations of the proposed phenomenological theory: The value $2a/d$ increases with decreasing value of Re_M to reach unity for $Re_M \approx 1{\cdot}0 \,.\, 10^3$. In view of the fact that a value of the source of the flow exceeding the radius of the impeller, $d/2$, has no physical meaning, this limit may be taken for the limit of validity of the proposed theory.

The effect of the size of the equipment on values of the discussed parameters of the stream may observed from comparison of σ and $A/(2\pi d^2 n)$ obtained in this work and in the earlier work of this series[12]. In the latter work we found on a geometrically similar system with $D = 1$ m ($d/D \in \langle 1/4; 1/3 \rangle$) analogous values $\sigma = 11{\cdot}2$; $A/(2\pi d^2 n) = 0{\cdot}120$, which is in good agreement with the mean values of these parameters

TABLE IV

Field of Eddy Viscosity in the Stream Streaking from the Rotating Turbine Impeller

$d/D = 0{\cdot}20$			$d/D = 0{\cdot}40$			$d/D = 0{\cdot}50$		
Re_M	R	$\varepsilon \,.\, 10^4$ $m^2 s^{-1}$	Re_M	R	$\varepsilon \,.\, 10^4$ $m^2 s^{-1}$	Re_M	R	$\varepsilon \,.\, 10^4$ $m^2 s^{-1}$
$2{\cdot}72 \,.\, 10^4$	0·0833	1·87	$1{\cdot}00 \,.\, 10^5$	0·167	48·0[a]	$3{\cdot}00 \,.\, 10^4$	0·040	1·53
$1{\cdot}65 \,.\, 10^3$	0·0214	2·73	$1{\cdot}00 \,.\, 10^5$	0·500	32·5[a]	$3{\cdot}00 \,.\, 10^4$	0·300	4·14
$1{\cdot}65 \,.\, 10^3$	0·607	5·18	$2{\cdot}94 \,.\, 10^4$	0·025	1·26	$1{\cdot}65 \,.\, 10^3$	0·030	1·26
$1{\cdot}85 \,.\, 10^2$	0·536	3·53	$2{\cdot}94 \,.\, 10^4$	0·417	4·14	$1{\cdot}65 \,.\, 10^3$	0·450	4·88
$1{\cdot}85 \,.\, 10^2$	0·0357	3·36	$1{\cdot}10 \,.\, 10^3$	0·025	1·87	$1{\cdot}75 \,.\, 10^2$	0·158	3·29
$5{\cdot}60 \,.\, 10^1$	0·536	1·73	$1{\cdot}85 \,.\, 10^2$	0·025	3·78	$1{\cdot}75 \,.\, 10^2$	0·040	2·99
$5{\cdot}60 \,.\, 10^1$	0·0357	1·99	$1{\cdot}85 \,.\, 10^2$	0·583	3·28	$2{\cdot}94 \,.\, 10^1$	0·040	3·34
			$5{\cdot}54 \,.\, 10^1$	0·545	1·55	$2{\cdot}94 \,.\, 10^1$	0·500	1·25
			$5{\cdot}54 \,.\, 10^1$	0·025	2·35			

[a] $D = 1{\cdot}0$ m.

summarized in Table III. Other situation, however, occurs when comparing eddy viscosities (Table IV), where the size of the system markedly influences the magnitude of ε; the differences amount to as much as an order of magnitude for a change of the characteristic dimension of the system by a factor of 2·5. This fact indicates that at constant value of Re_M the intensity of turbulence increases very rapidly with the scale of the system. On the other hand, the increased kinematic viscosity of the charge virtually does not influence the magnitude of the eddy viscosity. Yet, the ratio ε/ν markedly decreases up to unity for $Re_M < 1{\cdot}0 \,.\, 10^3$ for the given geometrical configuration.

Thus finding confirms the above conclusion about the limits of validity of the proposed description of the velocity field in the stream by the above proposed relations: If the viscous and turbulent stresses reach similar values in the order of magnitude they cannot be mutually neglected and the approach *via* modification of the Reynolds equation that has lead[5,6] to the suggested solution cannot be used.

Finally it is worth to note the option of computing the pumping capacity of the impeller $\dot{V}$ (the amount pumped by the impeller per unit time) from axial profiles of the radial velocity component in the stream streaking from the region of the rotating impeller. In dimensionless form this is so-called flow rate criterion in the form

$$K_p = \dot{V}/nd^3 \,. \tag{10}$$

The relation necessary for its evaluation is presented in Table I and values obtained from experimental data are shown in Table III. The found values of the quantity K_p indicate that it practically does not depend on the relative size of the impeller and vessel d/D. However, it reached only half of the values obtained from direct measurements of the velocity field in the examined stream[4]. This discrepancy may be accounted for by the simplification that in the underlying model the impeller is viewed as an axially symmetric tangential jet while the width of the emerging stream equals $s = 2r/\sigma$. In case of the volumetric flow rate through the impeller (here $r = d/2$), the width of the stream is less than one half of the height of the impeller blade. Hence the volumetric flow rate – the integral across its width – is lower than the volumetric flow rate through the impeller itself, *i.e.* the amount of liquid passing per unit time the jacket of the cylindrical surface made by the outer edges of the impeller blades. The obtained results also indicate a quantitatively detectable effect of the Reynolds number Re_M on the flow rate criterion K_p: for decreasing values of Re_M by an order of magnitude, K_p decreases by more than 10%. This result is worth noting for in this way the viscous forces play a certain, although not large, role in affecting the velocity field even in the automodel turbulent flow region.

LIST OF SYMBOLS

A universal parameter of the axial profile of the radial velocity component, $m^2 s^{-1}$
A_j $(j = 1, 2, 3)$ parameter of axial profile of radial mean velocity component
a radius of cylindrical tangential jet, m
b width of baffle, m
C constant in Eq. (8)
c exponent in Eq. (8)
D vessel diameter, m
d impeller diameter, m
d_1 diameter of disc of turbine impeller, m
H liquid height over bottom at rest, m
H_2 height of lower edge of blades over bottom, m
H'_2 height of disc of turbine impeller over bottom, m
h width of blade, m
$K_p = \dot{V}/(nd^3)$ flow rate criterion
L length of blade, m
N number of investigated axial profiles of mean velocity
R dimensionless radial coordinate
r radial coordinate, m
$s_{y,rel}$ relative deviation of quantity y
$Re_M = nd^2/\nu$ Reynolds criterion for mixing
$\dot{V}$ pumping capacity of the impeller, $m^3 s^{-1}$
W dimensionless mean velocity
$\bar{w}$ mean velocity, $m s^{-1}$
Z dimensionless axial coordinate
z axial coordinate, m
α_0 angular coordinate
α angle
β angle
ε eddy viscosity, $m^2 s^{-1}$
ν molecular kinematic viscosity, $m^2 s^{-1}$
σ universal parameter of axial profile of radial mean velocity component

Subscripts

ax axial
r radial
tg tangential

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